

Spin asymmetries in lepton-hadron and hadron-hadron collisions

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EXPERIMENTS

- Lepton-Hadron: CERN, DESY, JLab
- Hadron-Hadron: RHIC, FermiLab

Outline

- Introduction
- Transversity parton distribution in the Drell-Yan process
- Single spin asymmetries and p_T -dependent correlators
- T-odd parton densities and color gauge invariance
- Experimental evidence for single spin asymmetries
- Factorization and universality of p_T -dependent correlators
- Phenomenology of the Sivers effect
- Single spin asymmetries in inclusive DIS
- Summary

General strategy

1. Factorization

$$\sigma \propto (\text{pert. part}) \otimes (\text{non-pert. part}) + \mathcal{O}\left(\frac{1}{Q}\right)$$

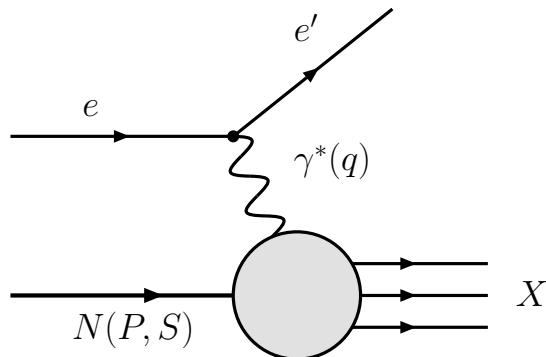
- required for interpretation
- does not hold for each hard process
- required scale: $Q > 1 \text{ GeV}$

2. Universality

- allows one to make predictions
- can be non-trivial

Inclusive deep-inelastic scattering

- Process



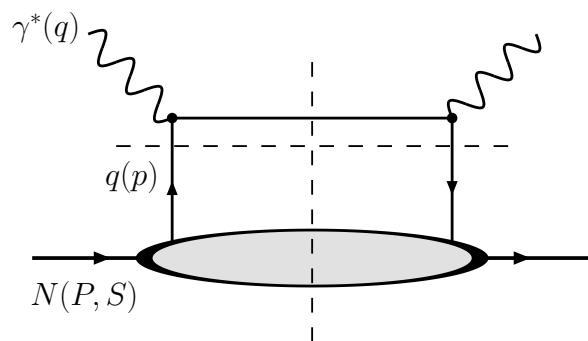
Variables

$$Q^2 = -q^2 > 0 \quad x = \frac{Q^2}{2P \cdot q} \quad \Phi_l^S$$

- Optical theorem

$$\sigma_{\gamma^* N \rightarrow X} \propto \text{Im } A(\gamma^* N \rightarrow \gamma^* N, \vartheta = 0)$$

- Parton model



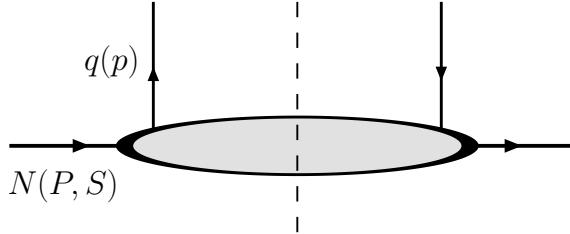
→ factorization

$$q(x, Q^2) \quad (f_1^q) \quad \Delta q(x, Q^2) \quad (g_1^q) \\ (g, \Delta g)$$

$$p^+ = \frac{1}{\sqrt{2}}(p^0 + p^3) = xP^+$$

Partonic functions

1. Forward parton densities



- Unpolarized pdf

$$f_1(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+ \xi^-} \langle P, S | \bar{\psi}(0) \gamma^+ \psi(\xi^-) | P, S \rangle$$

$$\xi^- = \frac{1}{\sqrt{2}}(t - z) \quad \xi^+ = \xi_T = 0$$

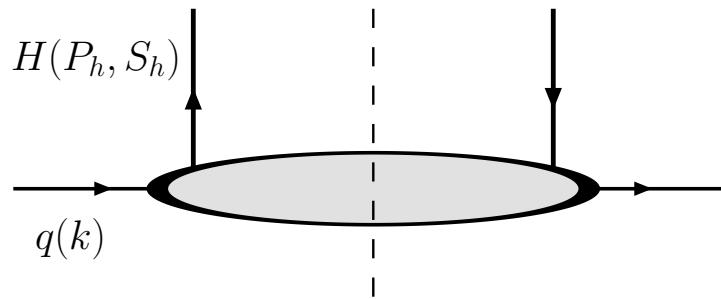
- Helicity pdf

$$g_1(x) \propto \langle | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(\xi^-) | \rangle$$

- Transversity pdf

$$h_1(x) \propto \langle | \bar{\psi}(0) i\sigma^{i+} \gamma_5 \psi(\xi^-) | \rangle \quad \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

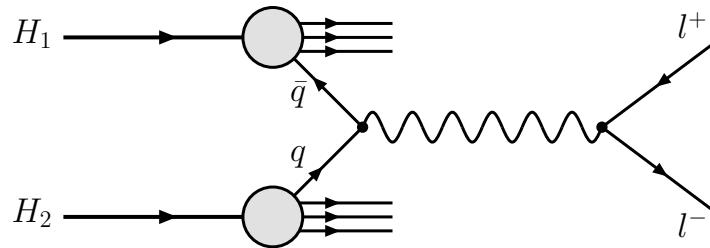
2. Forward fragmentation functions



$$\sum_X \langle 0 | \psi | H; X \rangle \Gamma \langle H; X | \bar{\psi} | 0 \rangle$$

Depending on $z = \frac{P_h^-}{k^-}$

Transversity in Drell-Yan



$$A_{TT}^{H_1 H_2} = a_{TT} \frac{\sum_q e_q^2 h_1^{q/H_1}(x_1, Q^2) h_1^{\bar{q}/H_2}(x_2, Q^2) + (x_1 \leftrightarrow x_2)}{\sum_q e_q^2 f_1^{q/H_1}(x_1, Q^2) f_1^{\bar{q}/H_2}(x_2, Q^2) + (x_1 \leftrightarrow x_2)}$$

- Kinematics

$$Q^2 \approx (x_1 P_1 + x_2 P_2)^2 \approx x_1 x_2 (P_1 + P_2)^2 = x_1 x_2 s$$

$$y = \frac{1}{2} \ln \frac{x_1}{x_2} \quad x_1 = \sqrt{\frac{Q^2}{s}} e^y \quad x_2 = \sqrt{\frac{Q^2}{s}} e^{-y} \quad x_F = x_1 - x_2$$

- Results

A_{TT}^{pp} small at RHIC ($\sqrt{s} = 200$ GeV) (Martin, Schäfer, Stratmann, Vogelsang, 1997)

A_{TT}^{pp} moderate at JPARC ($\sqrt{s} = 10$ GeV)

$A_{TT}^{p\bar{p}}$ large at GSI ($6 \text{ GeV} < \sqrt{s} < 15 \text{ GeV}$)

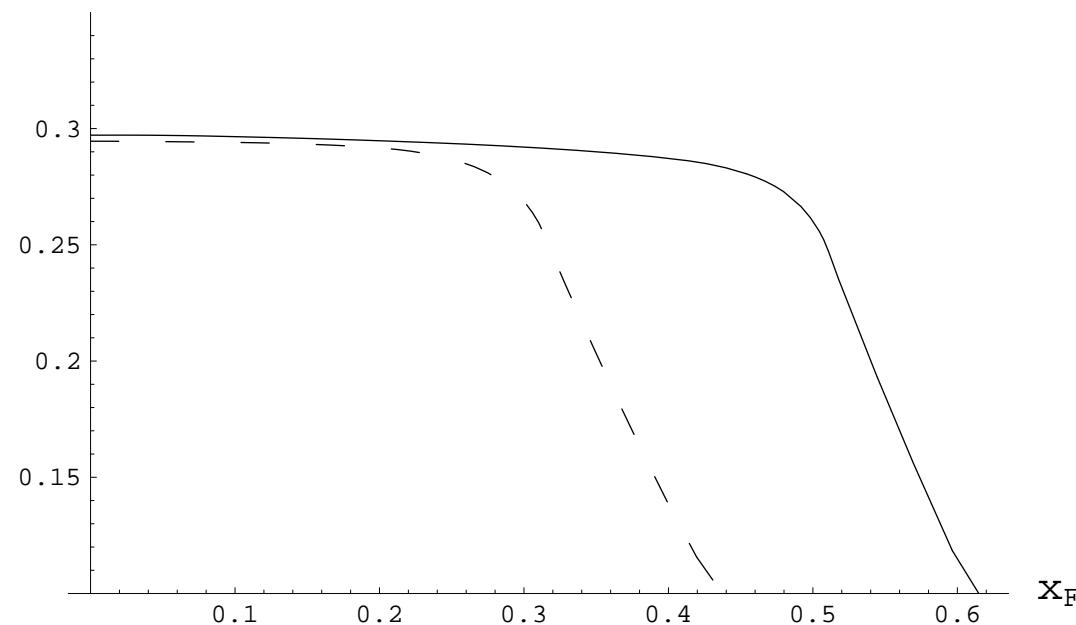
(Anselmino, Barone, Drago, Nikolaev, 2004)

$$s = 30 \text{ GeV}^2 \quad 45 \text{ GeV}^2$$

$$Q^2 = 16 \text{ GeV}^2$$

$$h_1^q(x, Q_0^2) = g_1^q(x, Q_0^2) \text{ at low scale } Q_0^2$$

$A_{\text{TT}}^{p \bar{p}} / a_{\text{TT}}$

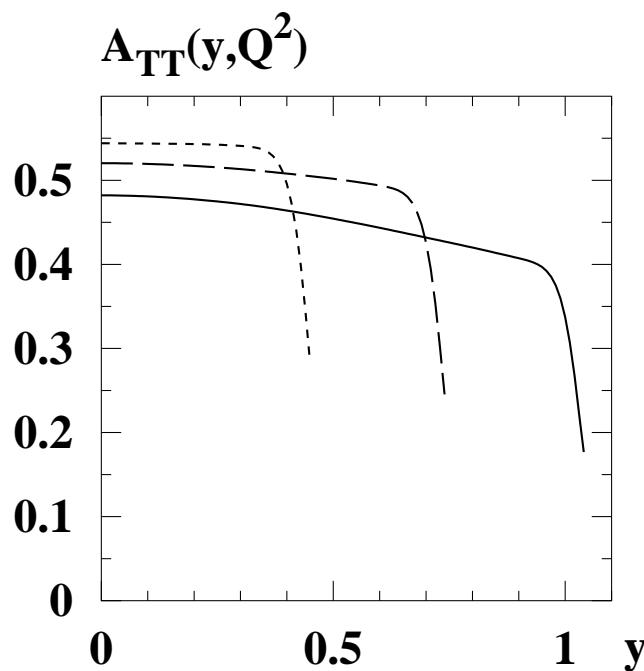


(Efremov, Goeke, Schweitzer, 2004)

$$s = 45 \text{ GeV}^2$$

$$Q^2 = 5 \text{ GeV}^2 \quad 9 \text{ GeV}^2 \quad 16 \text{ GeV}^2$$

$h_1^q(x, Q_0^2)$ from chiral quark soliton model



Classification of single spin asymmetries

1. Typ A

$$\sigma \propto A + \vec{S} \cdot \vec{P} B$$

- odd under parity
- even under (naïve) time-reversal
- Examples: $p \vec{p} \rightarrow e^- \bar{\nu}_e X$ ($\bar{u} \vec{d} \rightarrow W^- \rightarrow e^- \bar{\nu}_e$ on parton level)
 $\vec{e} p \rightarrow e p$ via exchange of Z

2. Typ B

$$\sigma \propto A + \vec{S} \cdot (\vec{P}_1 \times \vec{P}_2) B$$

- even under parity
- odd under naïve time-reversal
- requires two non-collinear momenta
- requires non-trivial phase on amplitude level

p_T -dependent parton densities

$$\begin{aligned}
 & \int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(xP^+ \xi^- - \vec{p}_T \cdot \vec{\xi}_T)} \langle P, S | \bar{\psi}(0) \gamma^+ \psi(\xi^-, \vec{\xi}_T) | P, S \rangle \\
 &= f_1(x, \vec{p}_T^2) - \frac{(\vec{p}_T \times \vec{S}_T) \cdot \hat{P}}{M} f_{1T}^\perp(x, \vec{p}_T^2)
 \end{aligned}$$

Parameterization of

$$\Phi^{[\Gamma]}(x, \vec{p}_T, S) = \int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(xP^+ \xi^- - \vec{p}_T \cdot \vec{\xi}_T)} \langle P, S | \bar{\psi}(0) \Gamma \psi(\xi^-, \vec{\xi}_T) | P, S \rangle$$

Leading order (8 functions, 2 T-odd)

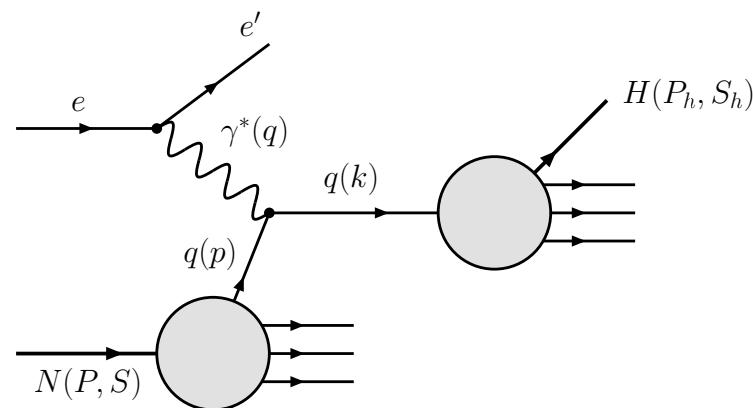
$$\begin{aligned}
 \Phi^{[\gamma^+]} &= f_1 - \frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^\perp \\
 \Phi^{[\gamma^+ \gamma_5]} &= \lambda g_{1L} + \frac{\vec{p}_T \cdot \vec{S}_T}{M} g_{1T} \\
 \Phi^{[i\sigma^{i+} \gamma_5]} &= S_T^i h_{1T}^\perp + \frac{p_T^i}{M} \left(\lambda h_{1L}^\perp + \frac{\vec{p}_T \cdot \vec{S}_T}{M} h_{1T}^\perp \right) - \frac{\varepsilon_T^{ij} p_{Tj}}{M} h_1^\perp
 \end{aligned}$$

Parton densities of the nucleon

	Quarks				Gluons			
Forward	f_1	g_1	h_1		g	Δg		
p_T -dependent	f_1	f_{1T}^\perp	g_{1L}	g_{1T}	g	Δg_L	g_T	Δg_T
	h_{1T}	h_{1L}^\perp	h_{1T}^\perp	h_1^\perp	h^\perp	Δh_T	Δh_L^\perp	Δh_T^\perp
Generalized	H	E	\tilde{H}	\tilde{E}	H^g	E^g	\tilde{H}^g	\tilde{E}^g
	H_T	E_T	\tilde{H}_T	\tilde{E}_T	H_T^g	E_T^g	\tilde{H}_T^g	\tilde{E}_T^g

p_T -dependent processes

1. $e N \rightarrow e' H X$



2. $H_1 H_2 \rightarrow l^+ l^- X$

3. $e^+ e^- \rightarrow H_1 H_2 X$

4. $H_1 H_2 \rightarrow jet_1 jet_2 X \quad H_1 H_2 \rightarrow jet H X \quad H_1 H_2 \rightarrow jet \gamma X$

T-odd parton densities

- (Sivers, 1990)

$$\int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(xP^+ \xi^- - \vec{p}_T \cdot \vec{\xi}_T)} \langle P, \uparrow | \bar{\psi}(0) \gamma^+ \psi(\xi^-, \vec{\xi}_T) | P, \uparrow \rangle$$

$$= f_1(x, \vec{p}_T^2) - \frac{(\vec{p}_T \times \vec{S}_T) \cdot \hat{P}}{M} f_{1T}^\perp(x, \vec{p}_T^2)$$

→ single-spin/azimuthal asymmetry

- (Boer, Mulders, 1998)

$$h_1^\perp(x, \vec{p}_T^2) \propto \langle P, \text{unp} | \bar{\psi}(0) i\sigma^{i+} \gamma_5 \psi(\xi^-, \vec{\xi}_T) | P, \text{unp} \rangle$$

Time-reversal:

$$f_{1T}^\perp = -f_{1T}^\perp = 0 \quad h_1^\perp = 0$$

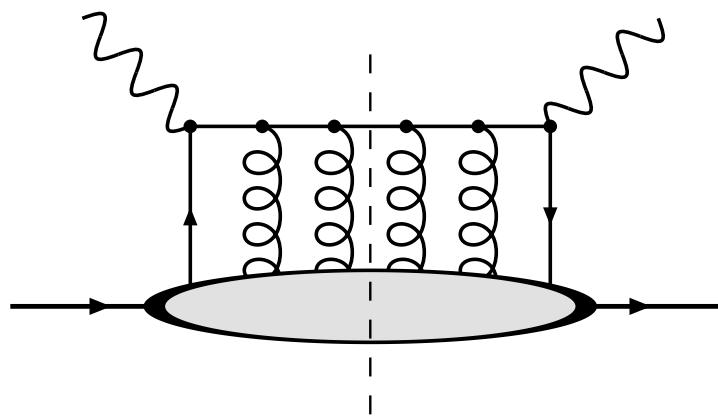
Color gauge invariance

1. Forward partonic functions

$$\int d\xi^- e^{ixP^+ \xi^-} \langle | \bar{\psi}(0) \Gamma \mathcal{L}(0; \xi^-) \psi(\xi^-) | \rangle$$

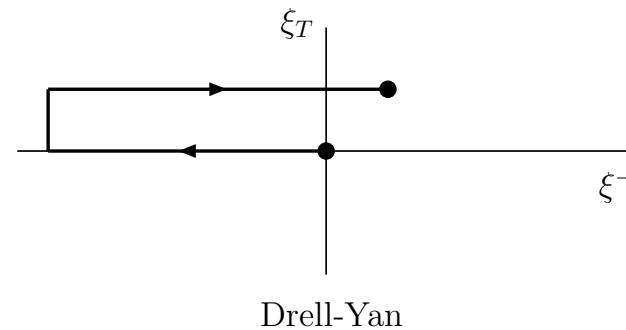
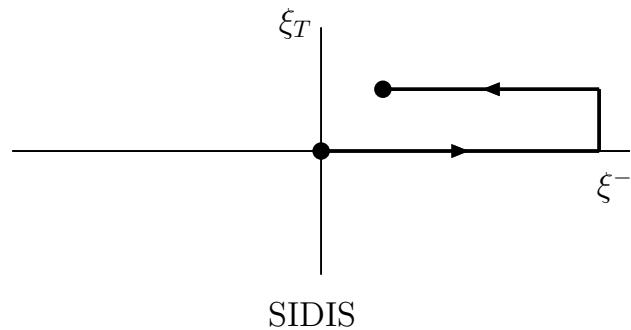
$$\mathcal{L}(0; \xi^-) = \mathcal{P} \exp \left(-ig \int_0^{\xi^-} d\eta^- A^+(0, \eta^-, \vec{0}_T) \right)$$

Gauge-link generated by rescattering



2. p_T -dependent partonic functions

$$\int d\xi^- d^2\vec{\xi}_T e^{i(xP^+ \xi^- - \vec{p}_T \cdot \vec{\xi}_T)} \langle | \bar{\psi}(0) \Gamma \mathcal{L}(0, \vec{0}_T; \xi^-, \vec{\xi}_T) \psi(\xi^-, \vec{\xi}_T) | \rangle$$

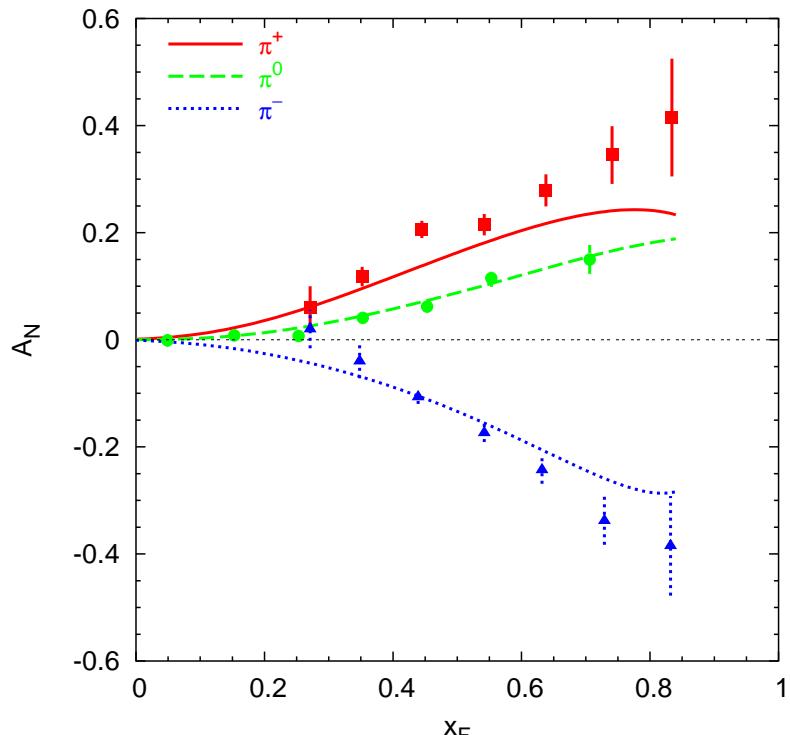


- $\mathcal{L}(0, \vec{0}_T; \xi^-, \vec{\xi}_T) = \mathcal{L}(0, \vec{0}_T; \infty, \vec{0}_T) \times \mathcal{L}(\infty, \vec{0}_T; \infty, \vec{\xi}_T) \times \mathcal{L}(\infty, \vec{\xi}_T; \xi^-, \vec{\xi}_T)$
(Belitsky, Ji, Yuan, 2002)
- Gauge-link → T-odd pdfs can be non-zero
(Brodsky, Hwang, Schmidt, 2002; Collins, 2002)
- Different links for semi-inclusive DIS and Drell-Yan → Universality?
Time-reversal: $f_{1T}^\perp|_{DY} = -f_{1T}^\perp|_{DIS}$ $h_1^\perp|_{DY} = -h_1^\perp|_{DIS}$
(Collins, 2002)

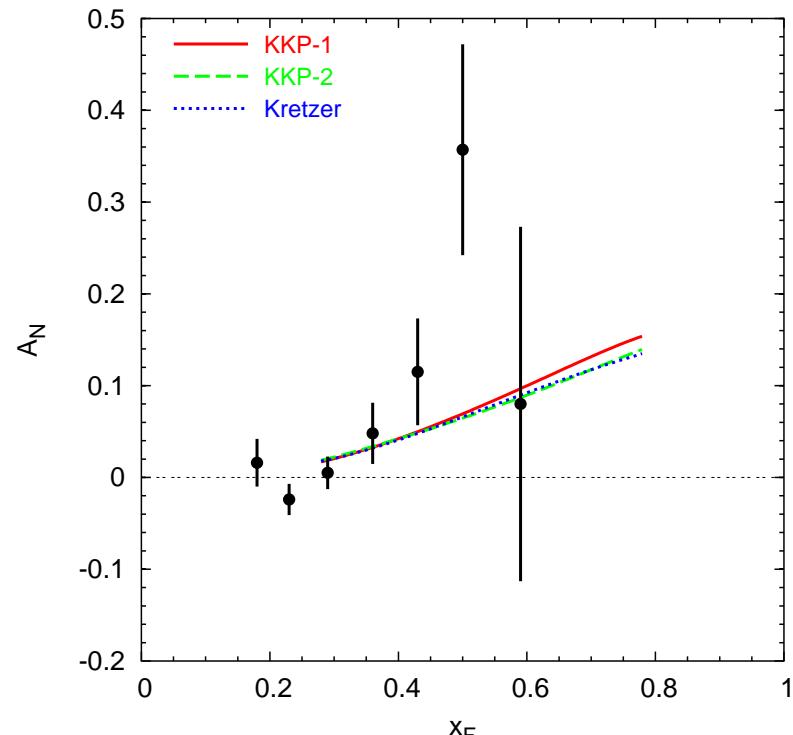
Transverse SSA in $p p^\uparrow \rightarrow \pi X$

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$x_F = \frac{2P_{hL}}{\sqrt{s}}$$

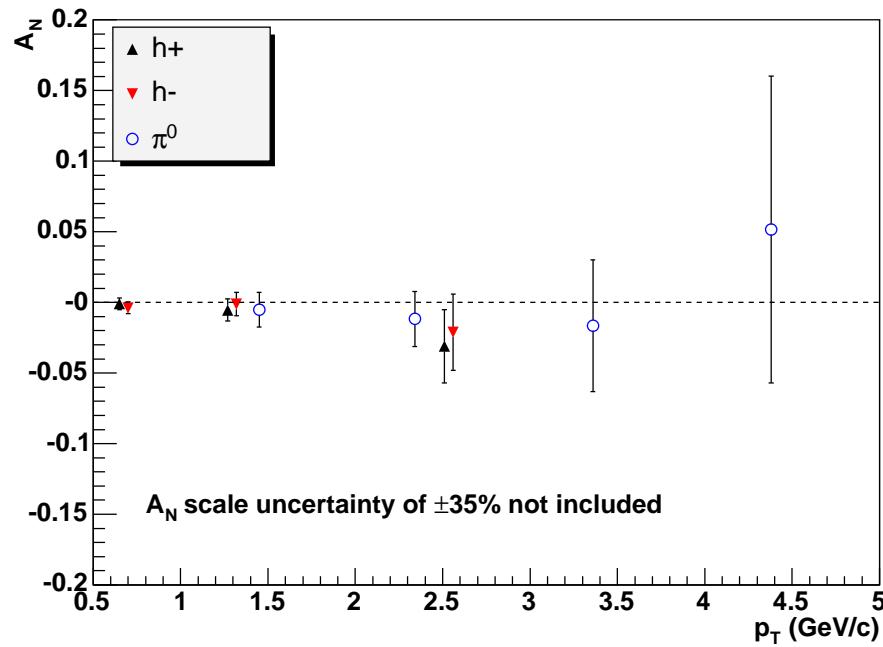


FermiLab, E704, 1990 $\sqrt{s} = 20 \text{ GeV}$



RHIC, STAR, 2004 $\sqrt{s} = 200 \text{ GeV}$

Also data from PHENIX and BRAHMS

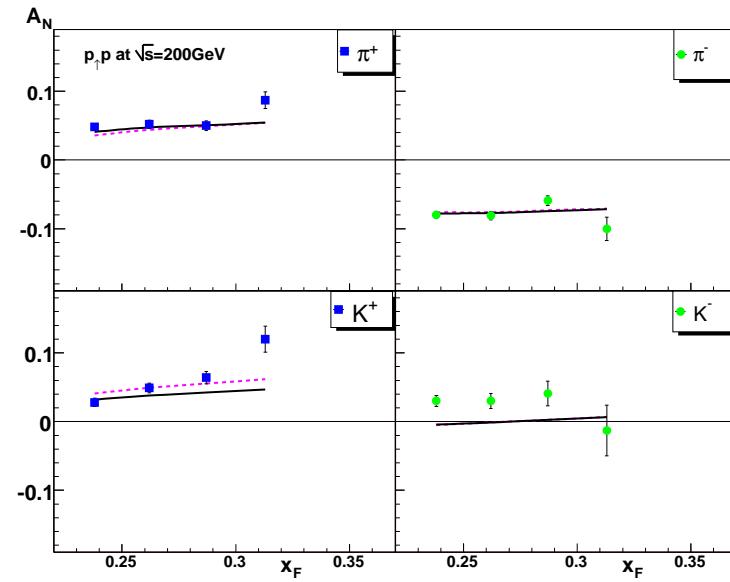
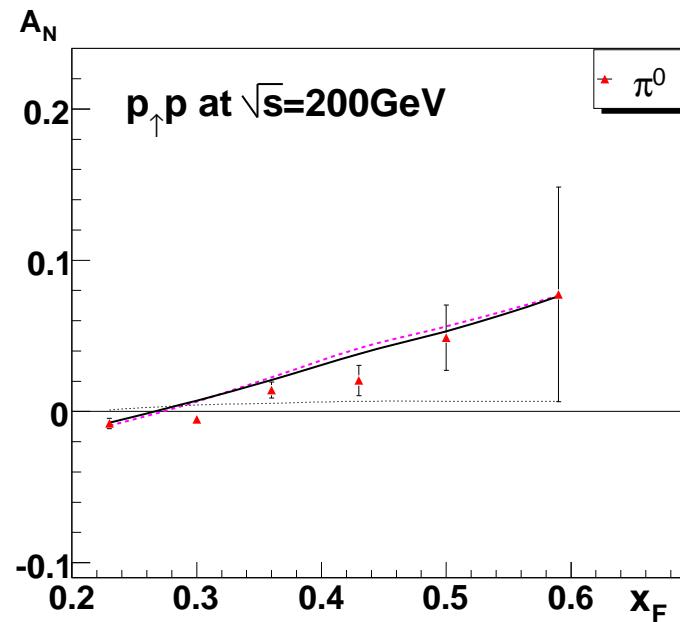


RHIC, PHENIX, 2005

Theory for $p p^\uparrow \rightarrow H X$

A_N vanishes in leading twist collinear pQCD-description

1. (Effective) p_T -dependent formalism
→ works well phenomenologically, but theory not really settled
2. Collinear twist-3 formalism
(Efremov, Teryaev, 1985; Qiu, Sterman, 1991;
Kouvaris, Qiu, Vogelsang, Yuan, 2006; etc.)
→ description in terms of matrix elements $\langle P, S | \bar{\psi} A_T \psi | P, S \rangle$ on the light-cone



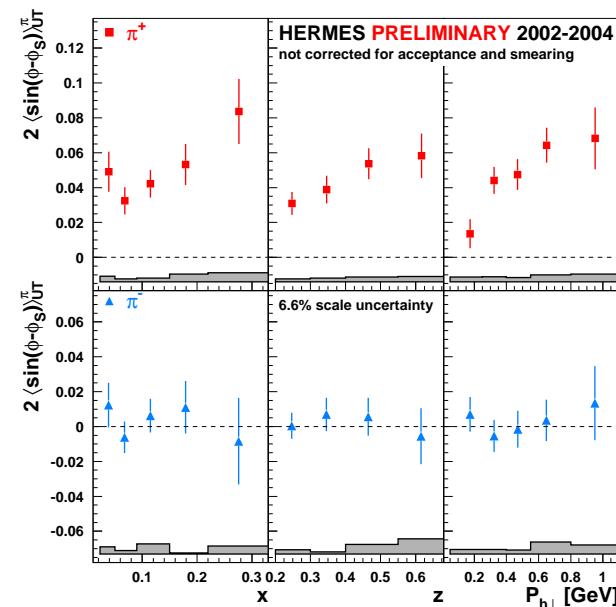
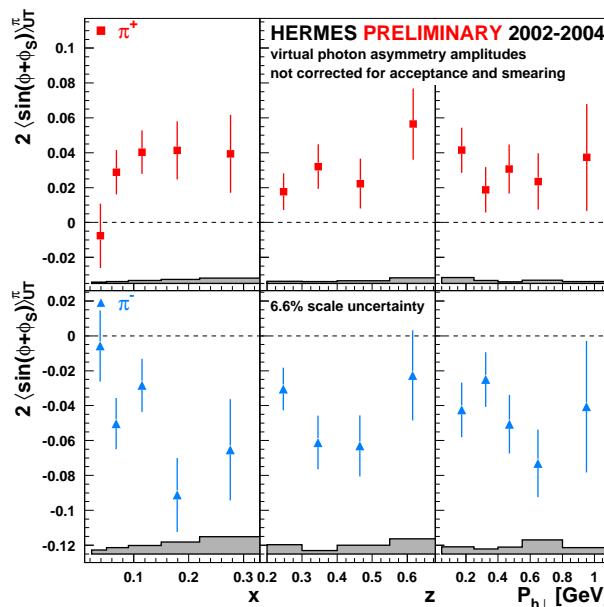
Plots from Kouvaris et al., 2006

SSAs in semi-inclusive DIS, $l p \rightarrow l H X$

1. Longitudinal SSAs (twist-3, $\propto 1/Q$)
 - Target spin asymmetry A_{UL} : data from HERMES
 - Beam spin asymmetry A_{LU} : data from CLAS

2. Transverse SSA (leading twist)

$$A_{UT} \propto \sin(\Phi_h + \Phi_S) h_1(x) H_1^\perp(z) + \sin(\Phi_h - \Phi_S) f_{1T}^\perp(x) D_1(z) \\ + \sin(3\Phi_h - \Phi_S) \dots + \mathcal{O}(1/Q)$$



Also COMPASS data on $l D^\dagger \rightarrow l' H X$

Semi-inclusive deep-inelastic scattering

$$\sigma_{UU} : \quad f_1(x) D_1(z) \quad \cos(2\Phi_h) h_1^\perp(x) H_1^\perp(z)$$

$$\sigma_{LL} : \quad g_{1L}(x) D_1(z)$$

$$\sigma_{LT} : \quad \cos(\Phi_h - \Phi_S) g_{1T}(x) D_1(z)$$

$$\sigma_{UL} : \quad \sin(2\Phi_h) h_{1L}^\perp(x) H_1^\perp(z)$$

$$\begin{aligned} \sigma_{UT} : \quad & \sin(\Phi_h - \Phi_S) f_{1T}^\perp(x) D_1(z) & \sin(\Phi_h + \Phi_S) h_1(x) H_1^\perp(z) \\ & \sin(3\Phi_h - \Phi_S) h_{1T}^\perp(x) H_1^\perp(z) \end{aligned}$$

→ Complete experiment for p_T -dependent parton densities

Drell-Yan process

$$\sigma_{UU} : \quad f_1(x_1) \, f_1(x_2) \quad h_1^\perp(x_1) \, h_1^\perp(x_2)$$

$$\sigma_{LL} : \quad g_{1L}(x_1) \, g_{1L}(x_2)$$

$$\sigma_{TT} : \quad h_1(x_1) \, h_1(x_2)$$

$$\sigma_{LT} : \quad g_{1L}(x_1) \, g_{1T}(x_2)$$

$$\sigma_{UL} : \quad h_1^\perp(x_1) \, h_{1L}^\perp(x_2)$$

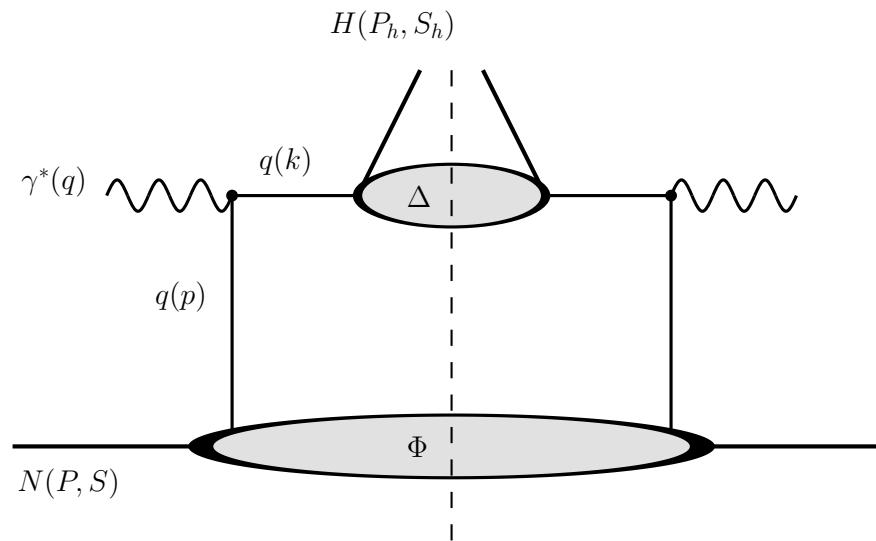
$$\sigma_{UT} : \quad f_1(x_1) \, f_{1T}^\perp(x_2) \quad h_1^\perp(x_1) \, h_1(x_2) \quad h_1^\perp(x_1) \, h_{1T}^\perp(x_2)$$

→ Complete experiment for p_T -dependent parton densities

Factorization of p_T -dependent correlators

1. Tree level

(Ralston, Soper, 1979)

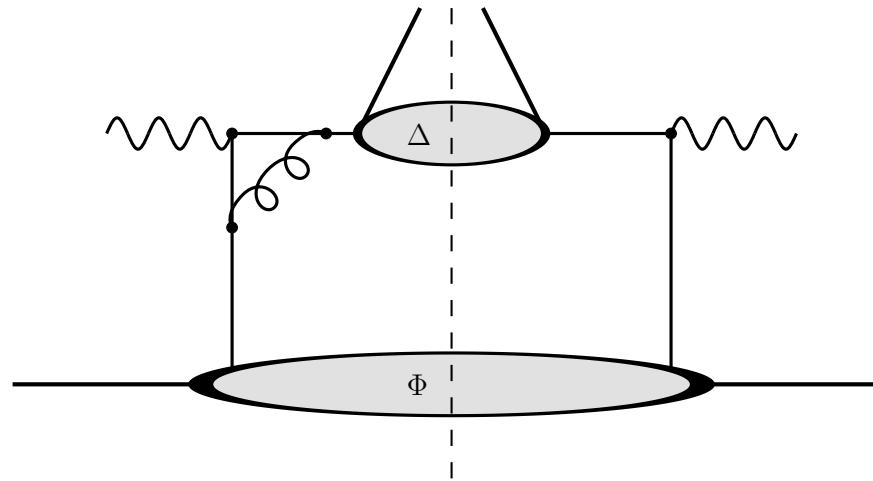


$$\frac{d\sigma_{unp}}{d^3 \vec{l}' d^3 \vec{P}_h} \propto \int d^2 \vec{p}_T d^2 \vec{k}_T f_1(x, \vec{p}_T^2) D_1(z, \vec{k}_T^2) \delta^{(2)}(\vec{p}_T + \vec{q}_T - \vec{k}_T) + \dots$$

$$f_1(x, \vec{p}_T^2) = \int \frac{d\xi^- d^2 \vec{\xi}_T}{2(2\pi)^3} e^{i(x P^+ \xi^- - \vec{p}_T \cdot \vec{\xi}_T)} \langle P, S | \bar{\psi}(0) \gamma^+ \psi(\xi^-, \vec{\xi}_T) | P, S \rangle$$

2. Beyond tree level

(Collins, Soper, 1981; Collins, Soper, Sterman, 1985;
 Ji, Ma, Yuan, 2004; Collins, Metz, 2004)



$$\frac{d\sigma_{unp}}{d^3 \vec{l}' d^3 \vec{P}_h} \propto \int d^2 \vec{p}_T d^2 \vec{k}_T d^2 \vec{l}_T f_1(x, \vec{p}_T^2) D_1(z, \vec{k}_T^2) \\ \times S(\vec{l}_T) H \delta^{(2)}(\vec{p}_T + \vec{q}_T + \vec{l}_T - \vec{k}_T) + \dots$$

- compatible with tree level result
- checked explicitly to $\mathcal{O}(\alpha_s)$ (Ji, Ma, Yuan, 2004)
- independent check through matching with collinear factorization at high P_{hT} (Ji, Qiu, Vogelsang, Yuan, 2006)
- arguments for consistency to all orders

3. Beyond leading twist

Relevant, e.g., for

Target spin asymmetry A_{UL} : data from HERMES

Beam spin asymmetry A_{LU} : data from CLAS

- Classification

(Mulders, Tangerman, 1995; Afanasev, Carlson, 2003;
Bacchetta, Mulders, Pijlman, 2004; Goeke, Metz, Schlegel, 2005; etc.)

Parameterization of

$$\Phi^{[\Gamma]}(x, \vec{p}_T, S) = \int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(xP^+ \xi^- - \vec{p}_T \cdot \vec{\xi}_T)} \langle P, S | \bar{\psi}(0) \Gamma \mathcal{L} \psi(\xi^-, \vec{\xi}_T) | P, S \rangle$$

- Factorization

→ no established factorization formalism for twist-3 p_T -dependent observables
like A_{UL} , A_{LU} , etc.

(Gamberg, Hwang, Metz, Schlegel, 2006)

Universality of p_T -dependent correlators

1. Problem

- Semi-inclusive DIS

$$\sigma|_{DIS} \propto \hat{\sigma}_{pert} \otimes \text{pdf} \otimes \text{ff} \otimes \text{soft}$$

- Drell-Yan

$$\sigma|_{DY} \propto \hat{\sigma}_{pert} \otimes \text{pdf} \otimes \text{pdf} \otimes \text{soft}$$

- $e^+ e^- \rightarrow H_1 H_2 X$

$$\sigma|_{e^+ e^-} \propto \hat{\sigma}_{pert} \otimes \text{ff} \otimes \text{ff} \otimes \text{soft}$$

$$\text{pdf}|_{DIS} \stackrel{?}{=} \text{pdf}|_{DY}$$

$$\text{ff}|_{DIS} \stackrel{?}{=} \text{ff}|_{e^+ e^-}$$

$$\text{soft}|_{DIS} \stackrel{?}{=} \text{soft}|_{DY} \stackrel{?}{=} \text{soft}|_{e^+ e^-}$$

2. Results

(Collins 2002; Metz 2002; Collins, Metz, 2004)

- Parton densities:

Time-reversal: $f_{1T}^\perp|_{DY} = -f_{1T}^\perp|_{DIS}$ $h_1^\perp|_{DY} = -h_1^\perp|_{DIS}$
6 T-even pdfs are universal

- Fragmentation functions:

Fragmentation functions in DIS and e^+e^- *a priori* have different links,
time-reversal cannot be applied

Analytical structure: $\text{ff}|_{DIS} = \text{ff}|_{e^+e^-}$ (for all 8 fragmentation functions)

- Soft factor:

Time-reversal and analytical structure: $\text{soft}|_{DIS} = \text{soft}|_{DY} = \text{soft}|_{e^+e^-}$

Phenomenology of the Sivers effect

(Collins, Efremov, Goeke, Grosse-Perdekamp, Menzel, Meredith,
Metz, Schweitzer, 2004, 2005)

Focus on Sivers effect in semi-inclusive DIS and Drell-Yan

1. Extraction of Sivers function f_{1T}^\perp from data on A_{UT} ($e p^\uparrow \rightarrow e \pi X$)

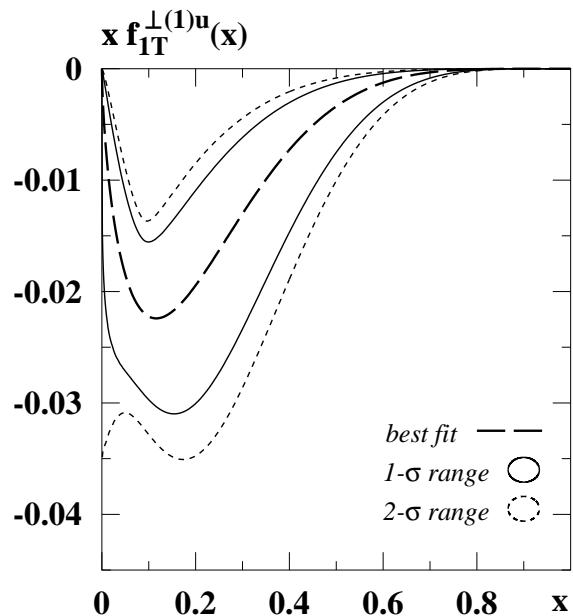
Input/approximations:

- HERMES data
- Neglect of soft gluon emission
- Gaussian p_T -behaviour, e.g.,

$$f_1(x, \vec{p}_T^2) = \frac{1}{\pi \langle p_T^2 \rangle} f_1(x) e^{-\vec{p}_T^2 / \langle p_T^2 \rangle}$$

- Exact result in large N_c -limit (Pobylitsa, 2003):

$$f_{1T}^{\perp u}(x, \vec{p}_T^2) = -f_{1T}^{\perp d}(x, \vec{p}_T^2)$$



$$f_{1T}^{\perp(1)}(x) \equiv \int d^2\vec{p}_T \frac{\vec{p}_T^2}{2M^2} f_{1T}^{\perp}(x, \vec{p}_T^2)$$

Extractions also by

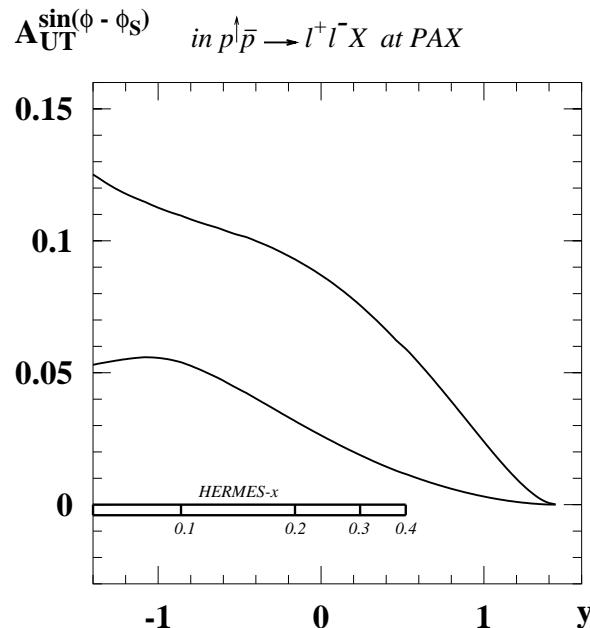
- Anselmino, Boglione, D' Alesio, Kotzinian, Murgia, Prokudin, 2005
- Vogelsang, Yuan, 2005

Comparison in: Anselmino et al., hep-ph/0511017

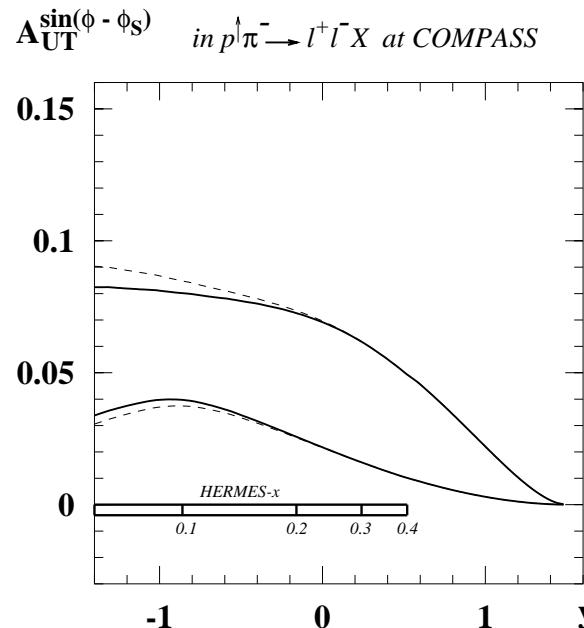
2. Calculation of the Sivers asymmetry for Drell-Yan

$$y = \frac{1}{2} \ln \frac{x_1}{x_2}$$

$$f_{1T}^{\perp(1)\bar{q}}(x) = \pm \frac{f_1^{\bar{u}}(x) + f_1^{\bar{d}}(x)}{f_1^u(x) + f_1^d(x)} f_{1T}^{\perp(1)q}(x)$$



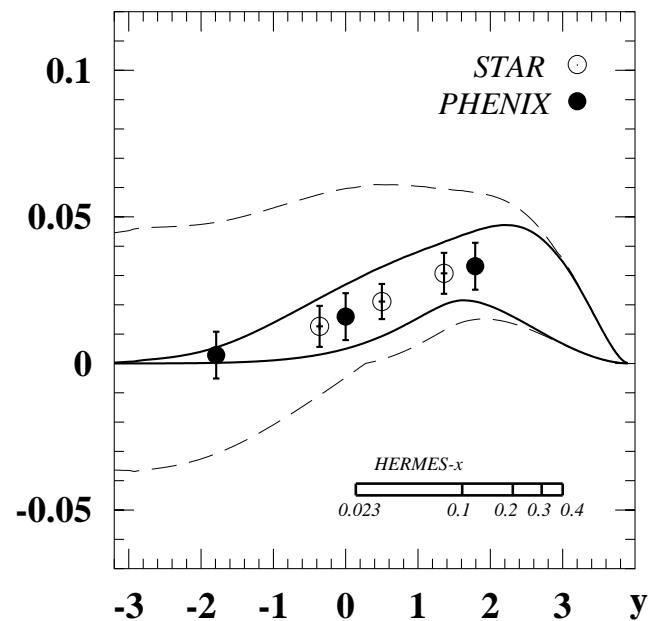
GSI $p^\uparrow \bar{p} \rightarrow l^+ l^- X$
 $s = 45 \text{ GeV}^2 \quad Q^2 = 6.25 \text{ GeV}^2$



COMPASS $p^\uparrow \pi^- \rightarrow l^+ l^- X$
 $s = 400 \text{ GeV}^2 \quad Q^2 = 20 \text{ GeV}^2$

→ Prediction $f_{1T}^\perp|_{DY} = -f_{1T}^\perp|_{DIS}$ can be checked experimentally

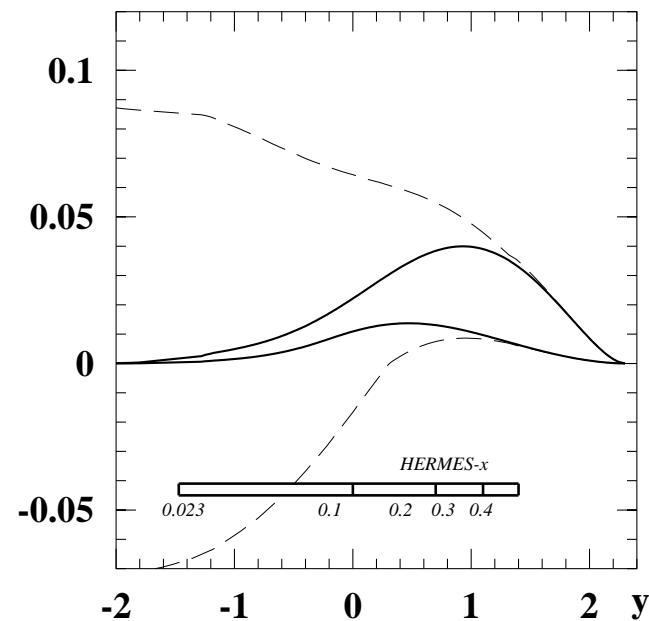
$A_{UT}^{\sin(\phi - \phi_S)}$ in $p^\uparrow p \rightarrow l^+ l^- X$ at RHIC $Q=4\text{GeV}$



RHIC $p^\uparrow p \rightarrow l^+ l^- X$

$\sqrt{s} = 200 \text{ GeV}$ $Q^2 = 16 \text{ GeV}^2$

$A_{UT}^{\sin(\phi - \phi_S)}$ in $p^\uparrow p \rightarrow l^+ l^- X$ at RHIC $Q=20\text{GeV}$



RHIC $p^\uparrow p \rightarrow l^+ l^- X$

$\sqrt{s} = 200 \text{ GeV}$ $Q^2 = 400 \text{ GeV}^2$

→ In pp -collisions strong sensitivity to $f_{1T}^{\perp \bar{q}}$

Single spin asymmetries in inclusive DIS

(Metz, Schlegel, Goeke, 2006)

1. One-photon exchange

- Unpolarized case

→ 2 structure functions: $F_1 \quad F_2$

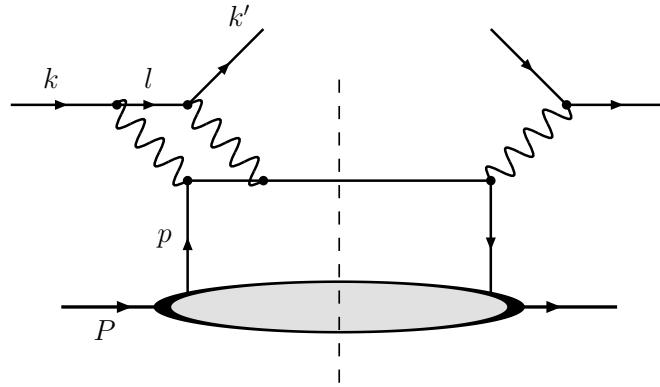
$$k'^0 \frac{d\sigma_{unp}}{d^3 \vec{k}'} = \frac{4 \alpha_{em}^2}{Q^4} \left(x y F_1(x, Q^2) + \frac{1-y}{y} F_2(x, Q^2) \right)$$

$$y = \frac{P \cdot (k - k')}{P \cdot k} = \frac{\nu_{Lab}}{E_{Lab}}$$

- (Double) polarized case

→ 2 structure functions: $g_1 \quad g_2$

2. Multi-photon exchange



- Lepton single spin asymmetry

$$k'^0 \frac{d\sigma_{L,pol}}{d^3 \vec{k}'} = \frac{4 \alpha_{em}^3}{Q^8} m x y^2 \varepsilon_{\mu\nu\rho\sigma} S^\mu P^\nu k^\rho k'^\sigma \sum_q e_q^3 x f_1^q(x)$$

- Target single spin asymmetry

$$\begin{aligned} k'^0 \frac{d\sigma_{N,pol}}{d^3 \vec{k}'} &= \frac{4 \alpha_{em}^3}{Q^8} \frac{M x^2 y}{1 - y} \varepsilon_{\mu\nu\rho\sigma} S^\mu P^\nu k^\rho k'^\sigma \sum_q e_q^3 x g_T^q(x) \\ &\times \left((1 - y)^2 \ln \frac{Q^2}{\lambda^2} + y(2 - y) \ln y + y(1 - y) \right) \end{aligned}$$

→ Requires inclusion of p_T -effects and of $\langle P, S | \bar{\psi} A_T \psi | P, S \rangle$

Summary

1. Spin asymmetries in lepton-hadron and hadron-hadron collisions is active field
→ information on h_1 , unintegrated correlation functions, twist-3 correlators
→ information on application of QCD
2. Leading p_T -dependent parton densities measurable in SIDIS, Drell-Yan,
 $p p \rightarrow jet_1 jet_2 X$ etc.
3. Factorization of p_T -dependent correlators
 - All-order factorization formula exists for
 $l N \rightarrow l H X$ $H_1 H_2 \rightarrow l^+ l^- X$ $e^+ e^- \rightarrow H_1 H_2 X$
 - Survives all available checks
 - Status of factorization beyond leading twist unclear
 - Situation more complicated for $p p \rightarrow H X$ $p p \rightarrow H_1 H_2 X$
(Bacchetta, Bomhof, Mulders, Pijlman, 2004, 2005, 2006; Vogelsang, Yuan,...)
4. Universality of p_T -dependent correlators
 - T-odd parton densities in DIS and Drell-Yan have reversed sign
 - Everything else is universal
5. Single spin asymmetry in inclusive DIS through multi-photon exchange

6. SSAs measured in processes like $p p^\uparrow \rightarrow \pi X$ and $p p^\uparrow \rightarrow jet_1 jet_2 X$
 \rightarrow information on twist-3 matrix elements, and on f_{1T}^\perp etc.

7. Different SSAs (A_{UL} , A_{LU} , A_{UT}) observed in $l p \rightarrow l H X$ and $l D \rightarrow l H X$
 \rightarrow information on h_1 , as well as f_{1T}^\perp and H_1^\perp , etc.

8. Angular distribution in unpolarized Drell-Yan
 \rightarrow information on h_1^\perp

9. Azimuthal $\cos 2\phi$ -asymmetry in $e^+ e^- \rightarrow H_1 H_2 X$ (Belle Collaboration)
 \rightarrow information on H_1^\perp

10. Future
 - Belle
 - COMPASS, HERMES, JLab (12 GeV-upgrade)
ELIC, ERHIC
 - RHIC, RHIC-II (pp)
 - JPARC (pp , $\sqrt{s} \simeq 10$ GeV), U70 (pp , $\sqrt{s} \simeq 12$ GeV)
 - FAIR@GSI ($\bar{p}p$, $\sqrt{s} \simeq 15$ GeV)
 - \rightarrow dedicated Drell-Yan measurement would allow one to check

$$f_{1T}^\perp|_{DY} = -f_{1T}^\perp|_{DIS} \quad h_1^\perp|_{DY} = -h_1^\perp|_{DIS}$$
 - etc.